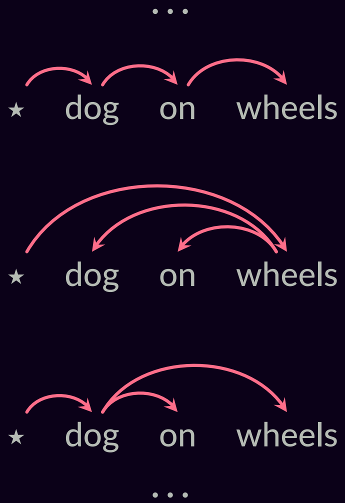
Learning with Sparse Latent Structure

Vlad Niculae
Instituto de Telecomunicações

Work with: André Martins, Claire Cardie, Mathieu Blondel

github.com/vene/sparsemap @vnfrombucharest

Structured Prediction



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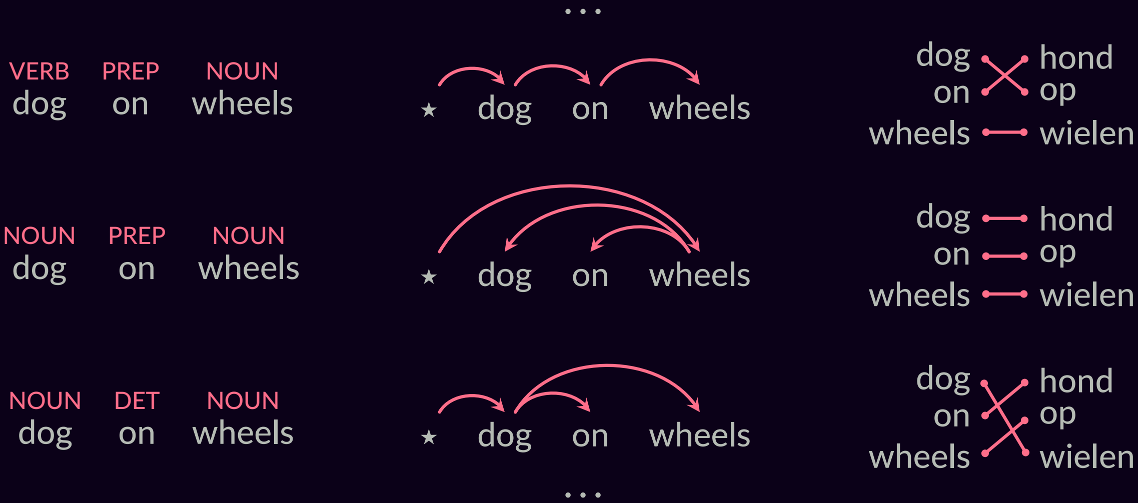

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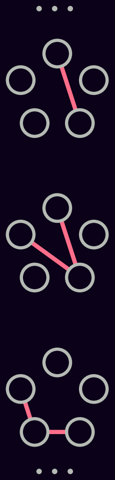
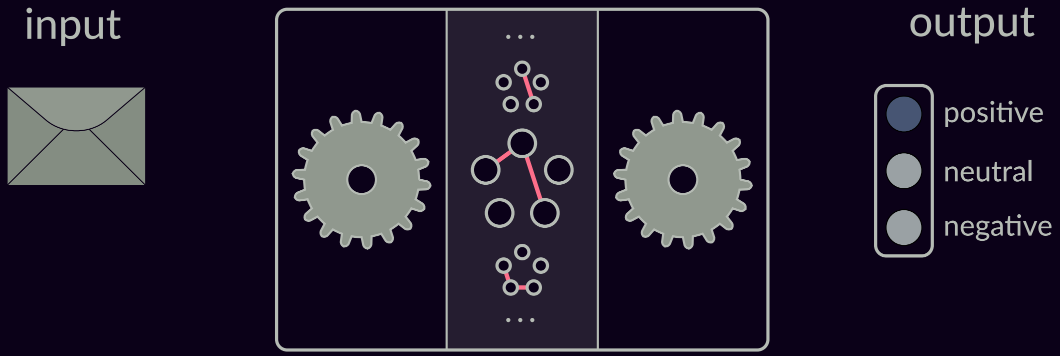


Latent Structure Models



record scratch

freeze frame

How to select an item from a set?

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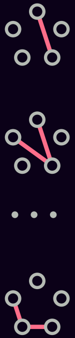

How to select an item from a set?

$$c_1$$

$$c_2$$

$$\cdots$$

$$c_N$$

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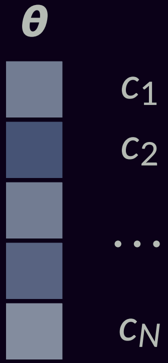

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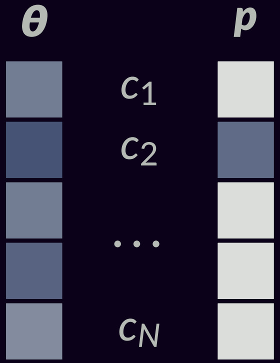

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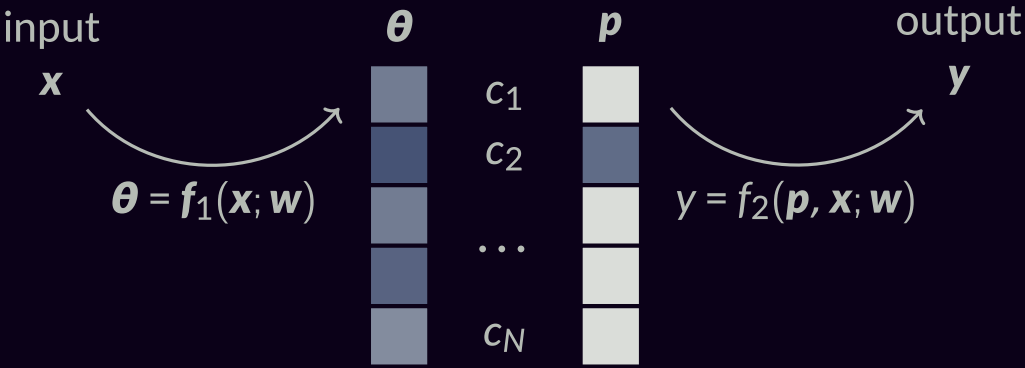

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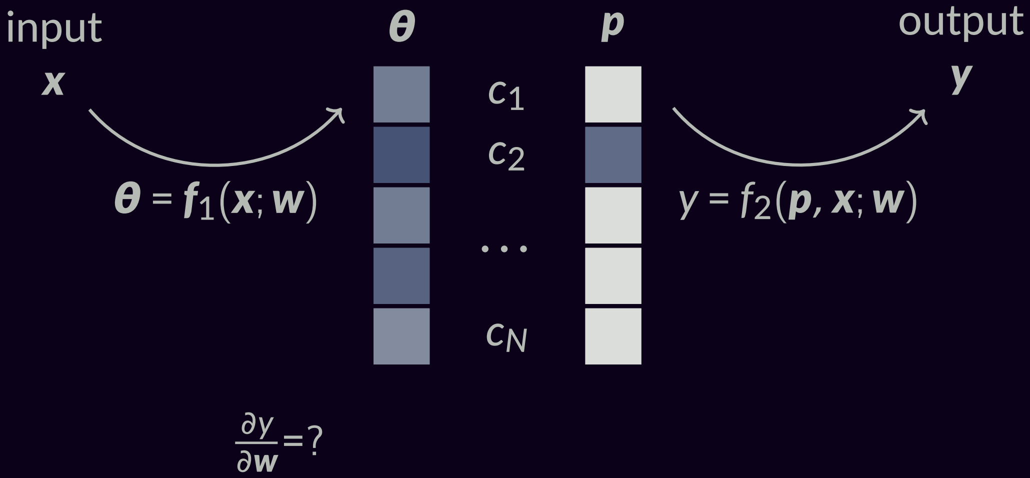

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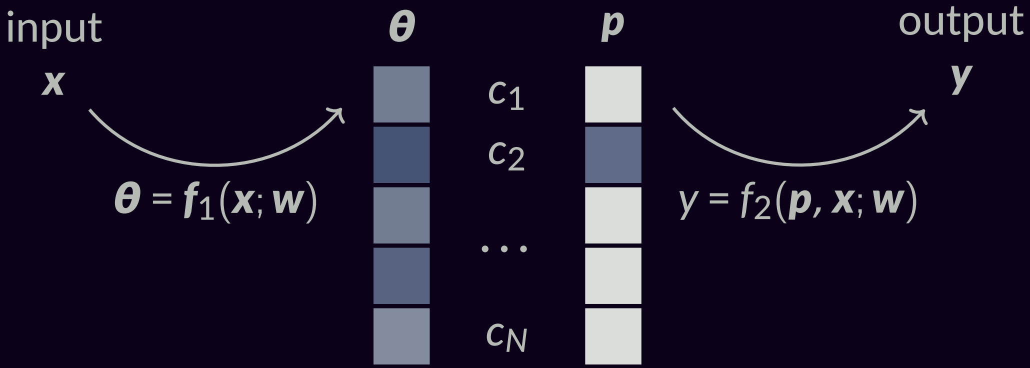
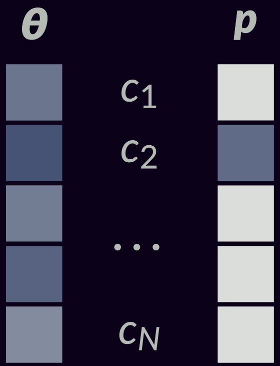


$\frac{\partial y}{\partial \boldsymbol{w}} = ?$ or, essentially, $\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}} = ?$

Argmax



$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}} = ?$$

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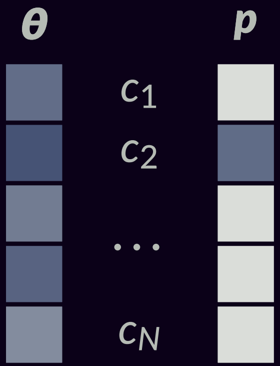

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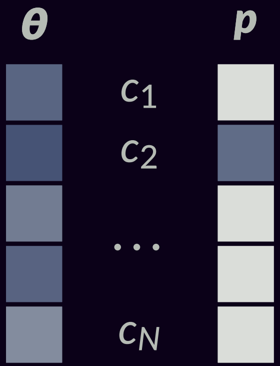

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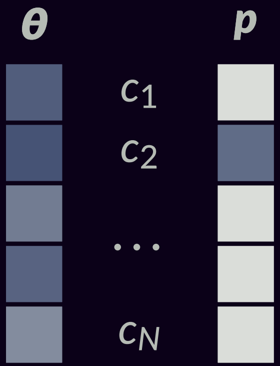

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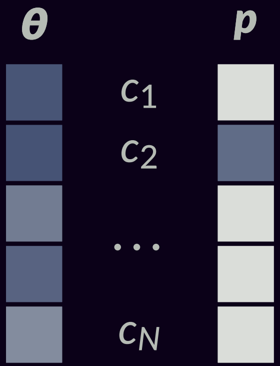

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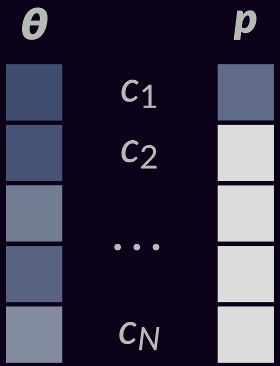

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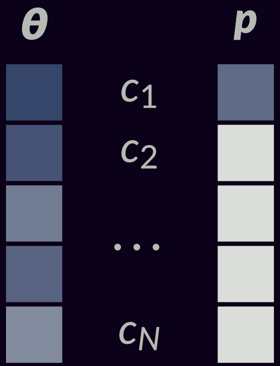

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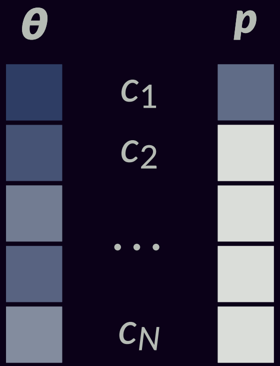

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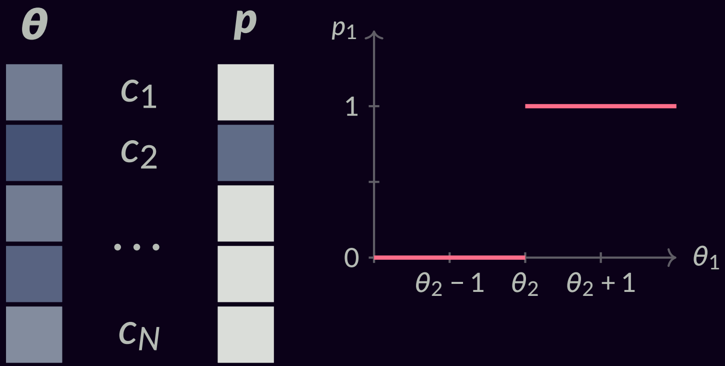
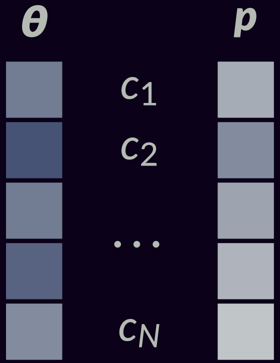
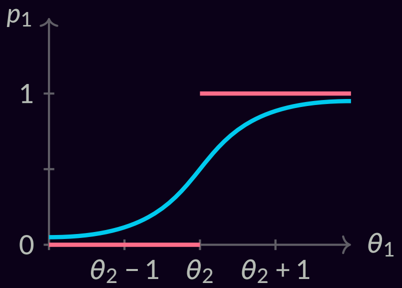


$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}} = \mathbf{0}$$

Argmax vs. Softmax

$$p_j = \exp(\theta_j)/Z$$





$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\theta}} = \mathrm{diag}(\boldsymbol{p}) - \boldsymbol{p}\boldsymbol{p}^\top$$

Variational Form of Argmax

$$\triangle = \{\boldsymbol{p} \in \mathbb{R}^N : \ \boldsymbol{p} \geq \mathbf{0}, \ \mathbf{1}^\top \boldsymbol{p} = 1\}$$

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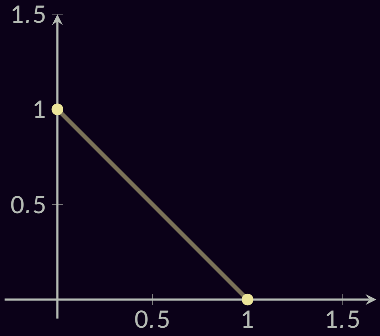

$N = 2$

Variational Form of Argmax

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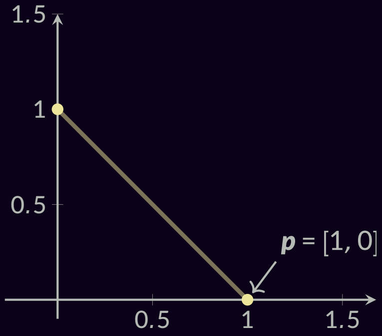

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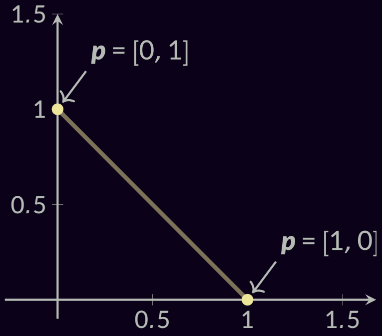

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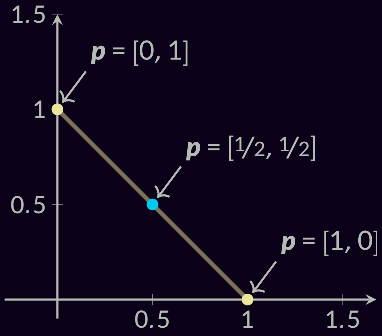

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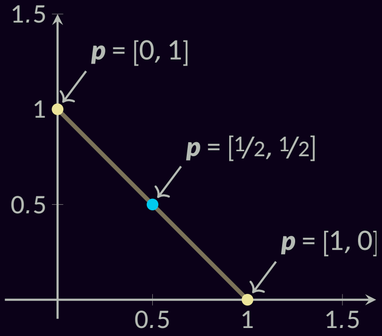

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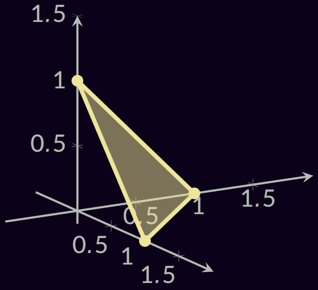

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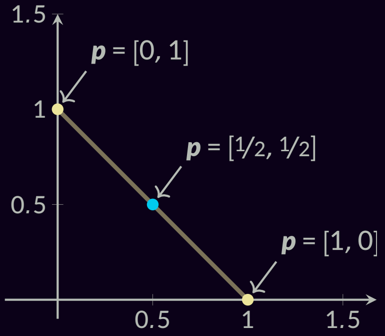

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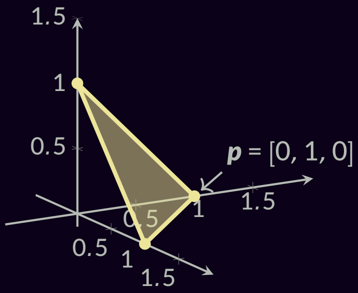

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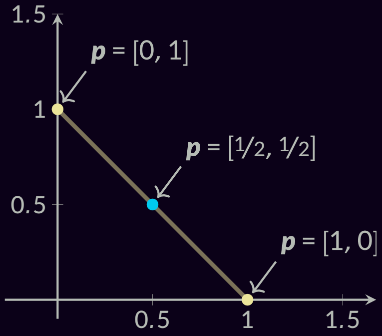

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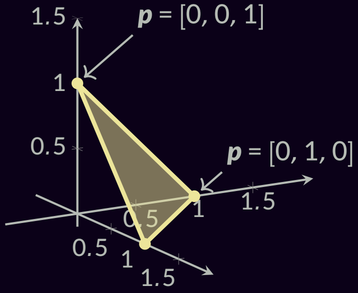

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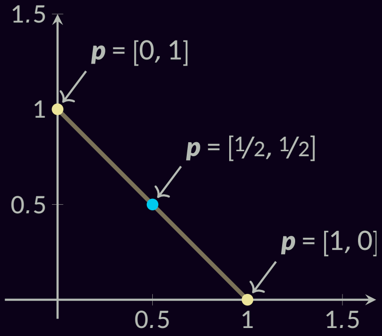

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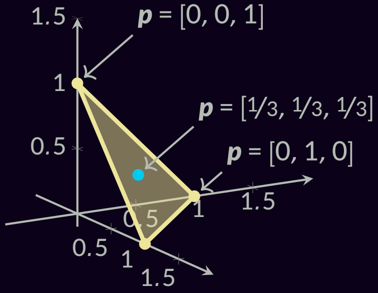

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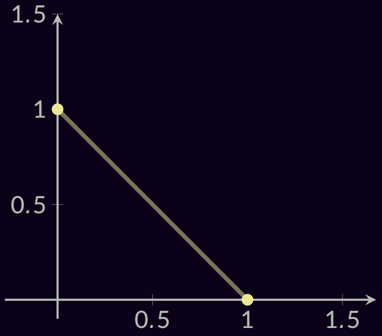
Variational Form of Argmax

$$\max_j \theta_j = \max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{\theta}$$

Fundamental Thm. Lin. Prog.
(Dantzig et al., 1955)



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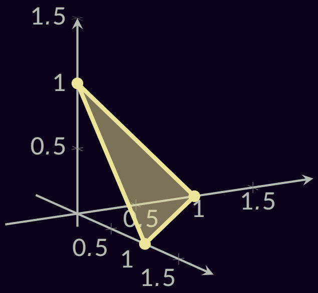

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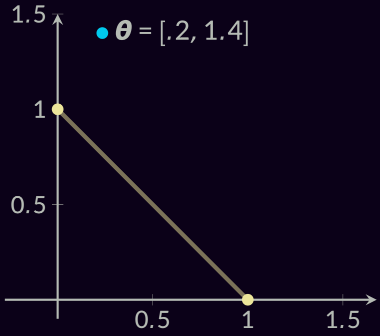

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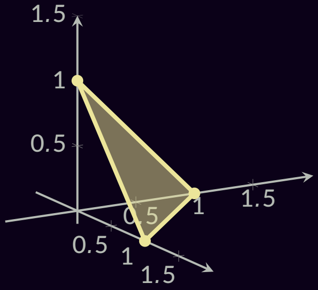

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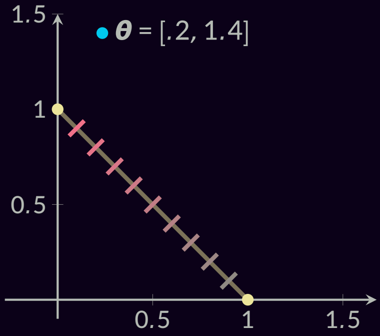

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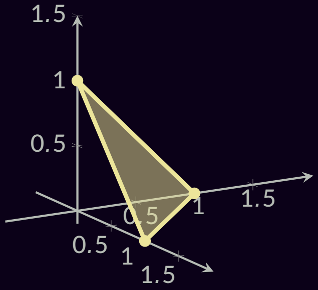

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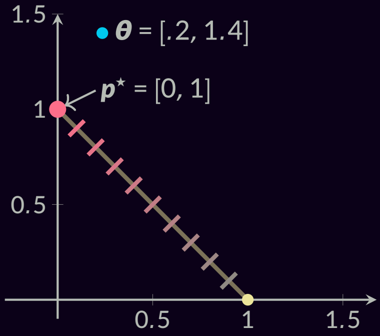

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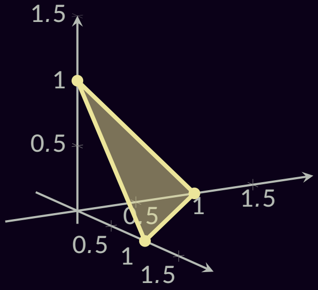

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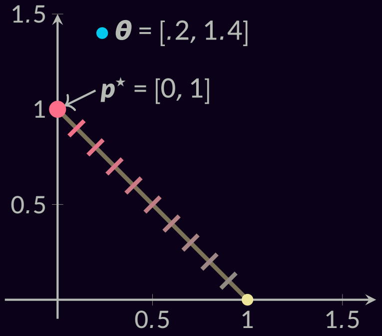

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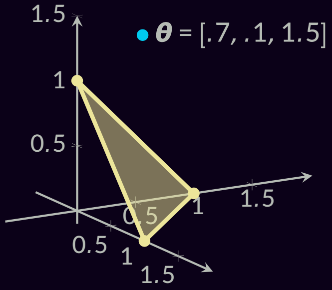

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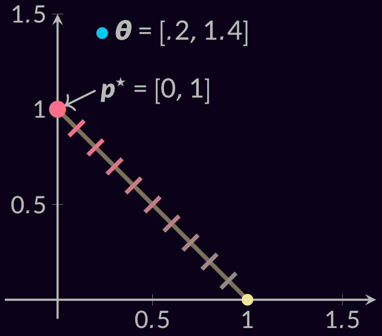

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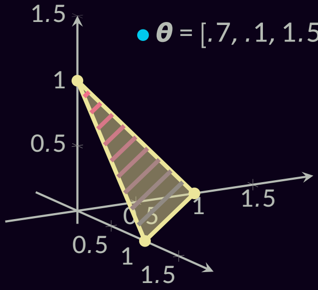

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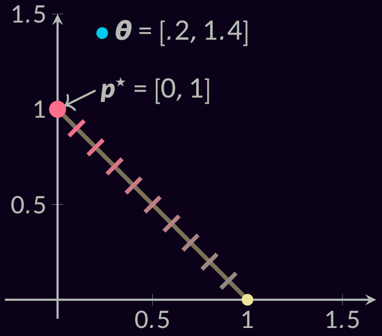

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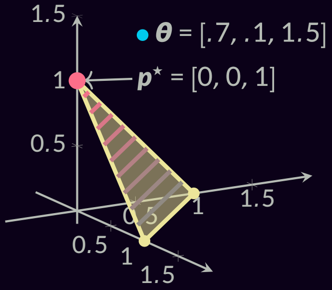

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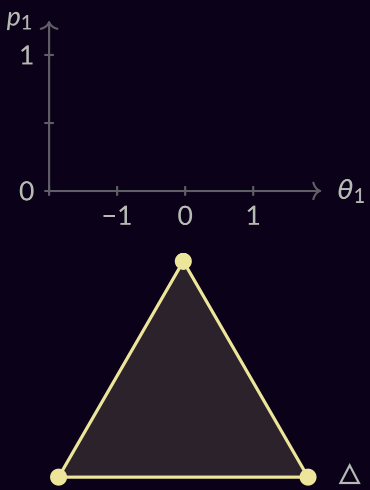
(Niculae and Blondel, 2017)

Smoothed Max Operators

$$\pi_\Omega(\boldsymbol{\theta}) = \arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{\theta} - \Omega(\boldsymbol{p})$$



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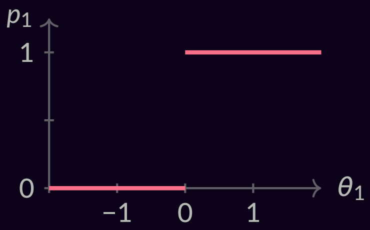

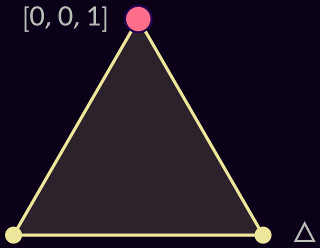

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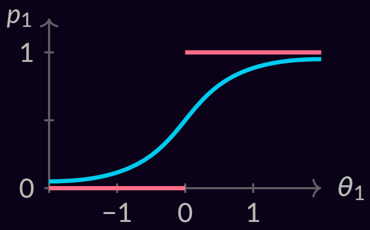
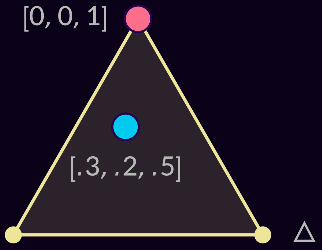
- argmax: $\Omega(\boldsymbol{p}) = 0$
- softmax: $\Omega(\boldsymbol{p}) = \sum_j p_j \log p_j$





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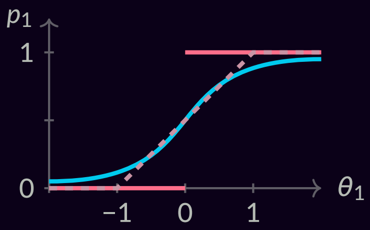
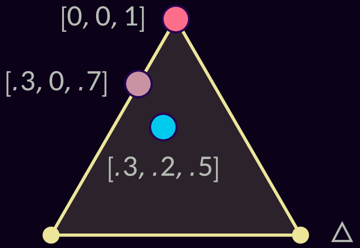
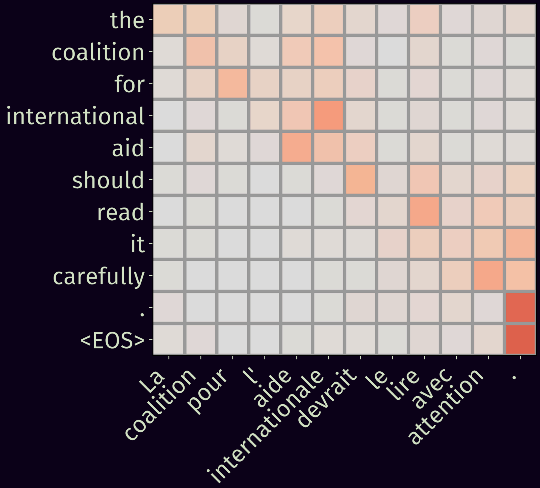
- argmax: $\Omega(\boldsymbol{p}) = 0$
- softmax: $\Omega(\boldsymbol{p}) = \sum_j p_j \log p_j$
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2 \|\boldsymbol{p}\|_2^2$





(Martins and Astudillo, 2016)



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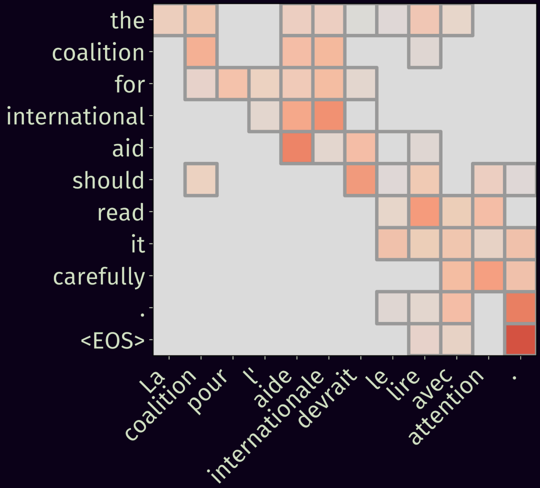
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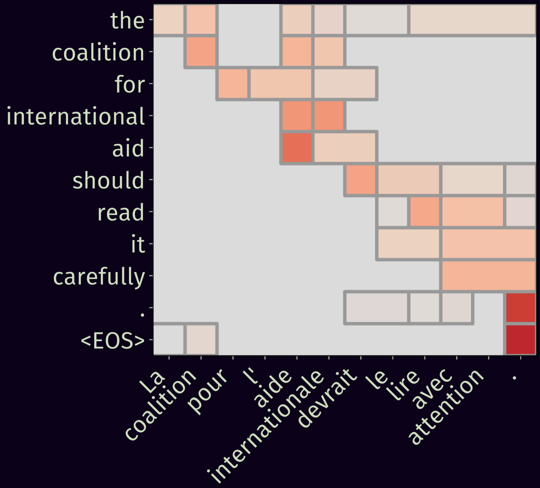

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<EOS>
La
coalition
pour
l'
aide
internationale
devrait
le
lire
avec
attention
.

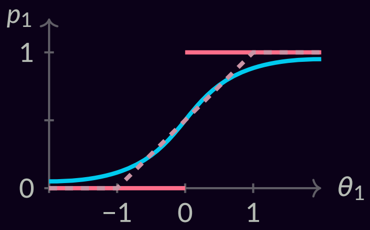
fusedmax ?!

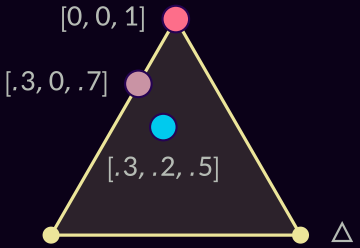
(Niculae and Blondel, 2017)

Smoothed Max Operators

$$\boldsymbol{\pi}_\Omega(\boldsymbol{\theta}) = \underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top\boldsymbol{\theta} - \Omega(\boldsymbol{p})$$

- argmax: $\Omega(\boldsymbol{p}) = 0$
- softmax: $\Omega(\boldsymbol{p}) = \sum_j p_j \log p_j$
- sparsemax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2$





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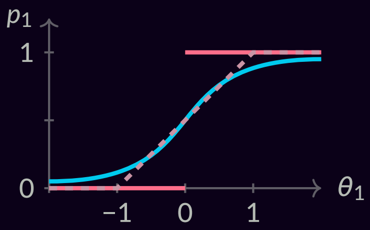
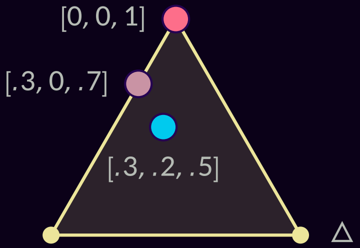
fusedmax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2 + \sum_j |p_j - p_{j-1}|$

csparsemax: $\Omega(\boldsymbol{p}) = 1/2\|\boldsymbol{p}\|_2^2 + \iota(\boldsymbol{a} \le \boldsymbol{p} \le \boldsymbol{b})$



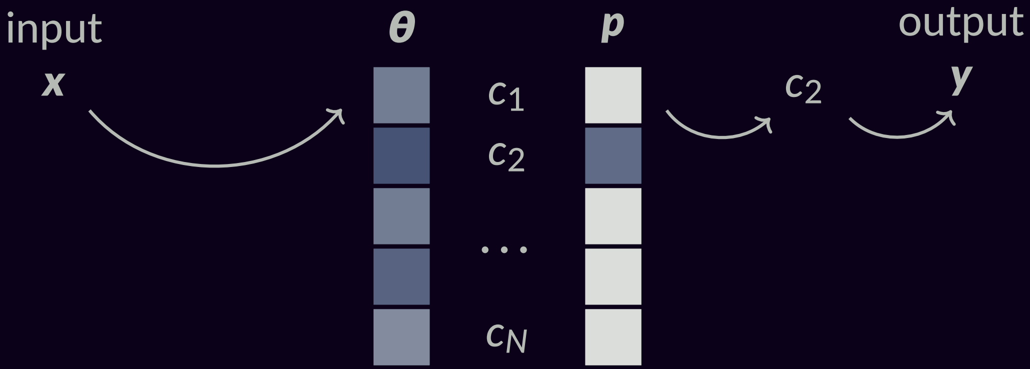


Structured Prediction

finally

Structured Prediction

is essentially a (very high-dimensional) argmax



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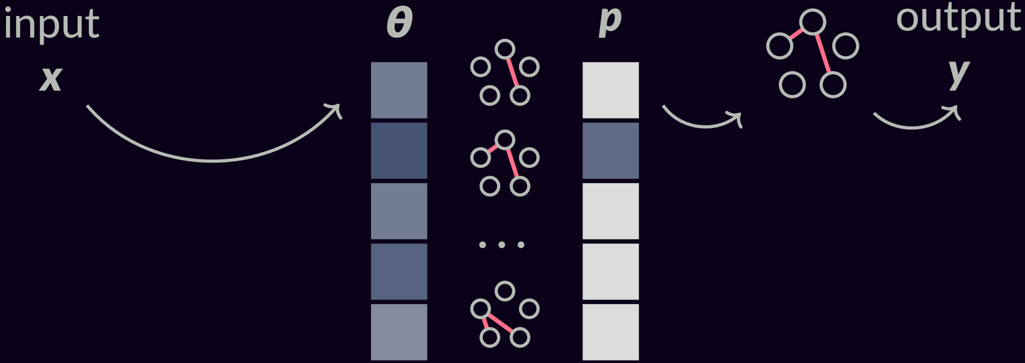

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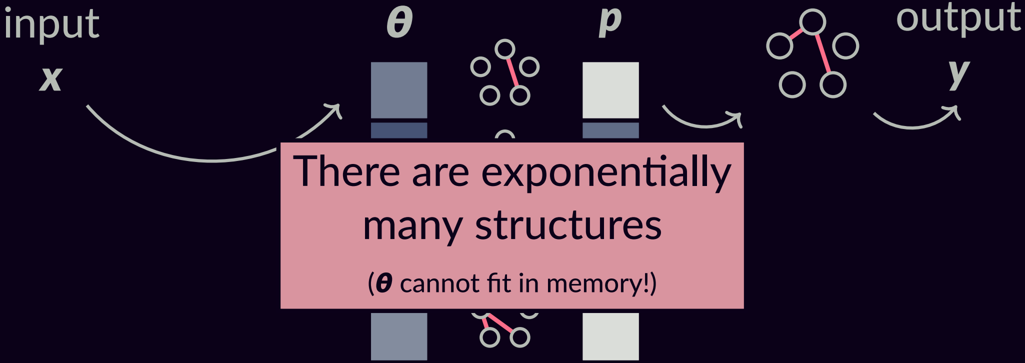

Factorization Into Parts

$$\boldsymbol{\theta} = A^\top \boldsymbol{\eta}$$

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$$\boldsymbol{\theta} = A^\top \boldsymbol{\eta}$$

★ dog on wheels

$$\boldsymbol{A} = \begin{array}{r} \star\to\text{dog} \\ \text{on}\to\text{dog} \\ \text{wheels}\to\text{dog} \\ \hline \star\to\text{on} \\ \text{dog}\to\text{on} \\ \text{wheels}\to\text{on} \\ \hline \star\to\text{wheels} \\ \text{dog}\to\text{wheels} \\ \text{on}\to\text{wheels} \end{array} \left[\begin{array}{ccccc} 1 & & 0 & 0 & \\ 0 & & 1 & 1 & \\ 0 & & 0 & 0 & \\ \hline 0 & & 1 & 1 & \\ 1 & \dots & 0 & 0 & \dots \\ 0 & & 0 & 0 & \\ \hline 0 & & 0 & 0 & \\ 0 & & 1 & 0 & \\ 1 & & 0 & 1 & \end{array}\right] \quad \boldsymbol{\eta} = \left[\begin{array}{r} .1 \\ .2 \\ -.1 \\ \hline .3 \\ .8 \\ .1 \\ \hline -.3 \\ .2 \\ -.1 \end{array}\right]$$

Factorization Into Parts

$$\boldsymbol{\theta} = A^\top \boldsymbol{\eta}$$

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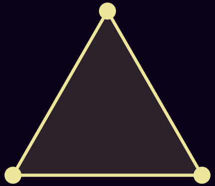
$$\boldsymbol{A} = \begin{array}{r} \text{dog—hond} \\ \text{dog—op} \\ \text{dog—wielen} \\ \text{on—hond} \\ \text{on—op} \\ \text{on—wielen} \\ \text{wheels—hond} \\ \text{wheels—op} \\ \text{wheels—wielen} \end{array} \left[\begin{array}{ccccc} 1 & & 0 & 0 & \\ 0 & & 1 & 1 & \\ 0 & & 0 & 0 & \\ \hline 0 & & 0 & 0 & \\ 1 & \cdots & 0 & 0 & \cdots \\ 0 & & 1 & 1 & \\ \hline 0 & & 1 & 0 & \\ 0 & & 0 & 0 & \\ 1 & & 0 & 1 & \end{array}\right] \quad \boldsymbol{\eta} = \left[\begin{array}{r} .1 \\ .2 \\ -.1 \\ \hline .3 \\ .8 \\ .1 \\ \hline -.3 \\ .2 \\ -.1 \end{array}\right]$$



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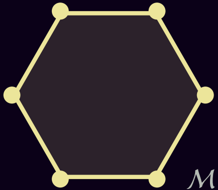
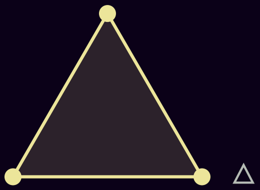
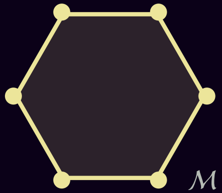


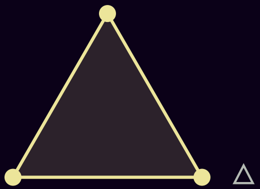
$\mathcal{M}$

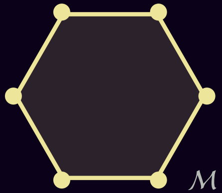
$$\mathcal{M} := \operatorname{conv}\left\{\boldsymbol{a}_y : y \in \mathcal{Y}\right\}$$



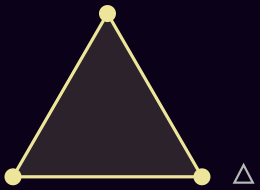


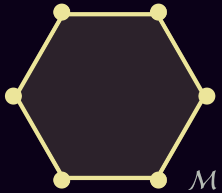
$$\mathcal{M} := \operatorname{conv}\left\{\boldsymbol{a}_y : y \in \mathcal{Y}\right\}$$
$$= \left\{\boldsymbol{A}\boldsymbol{p} : \boldsymbol{p} \in \triangle\right\}$$



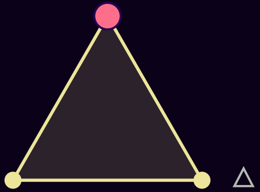


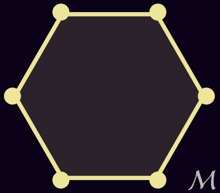
$$
\begin{aligned}
\mathcal{M} &:= \operatorname{conv}\left\{\boldsymbol{a}_y : y \in \mathcal{Y}\right\} \\
&= \left\{\boldsymbol{A}\boldsymbol{p} : \boldsymbol{p} \in \triangle\right\} \\
&= \left\{\mathbb{E}_{Y \sim \boldsymbol{p}}\, \boldsymbol{a}_Y : \boldsymbol{p} \in \triangle\right\}
\end{aligned}
$$





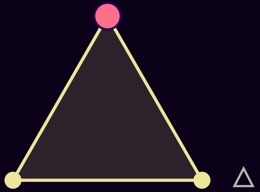
- **argmax** $\arg\max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{\theta}$

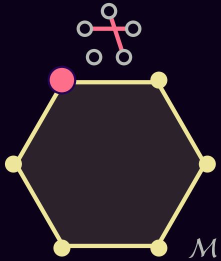




argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{\theta}$

MAP $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta}$





- **argmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{\theta}$
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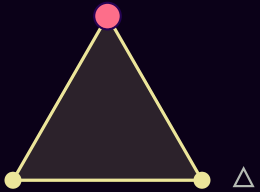
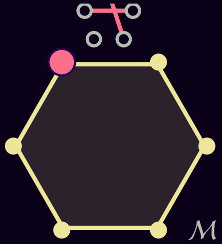
e.g. dependency parsing ⟶ **max. spanning tree**
matching ⟶ **the Hungarian algorithm**

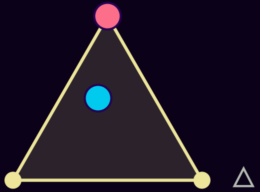


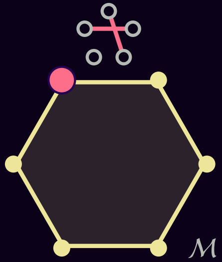


argmax $\operatorname*{arg\,max}_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top\boldsymbol{\theta}$

MAP $\operatorname*{arg\,max}_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top\boldsymbol{\eta}$

softmax $\operatorname*{arg\,max}_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top\boldsymbol{\theta} + \mathrm{H}(\boldsymbol{p})$





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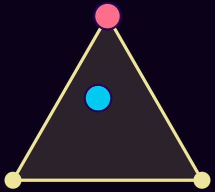
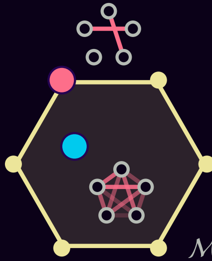
MAP $\displaystyle\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top\boldsymbol{\eta}$

marginals $\displaystyle\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top\boldsymbol{\eta} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$



$\triangle$



$\mathcal{M}$

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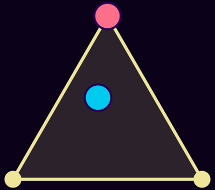
e.g. sequence labelling → forward-backward

(Rabiner, 1989)

As attention: (Kim et al., 2017)



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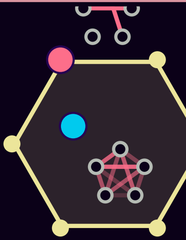

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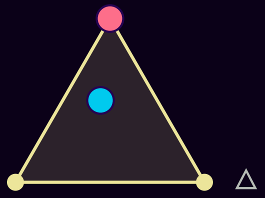
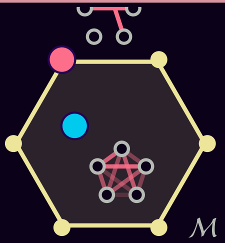
e.g. dependency parsing → **the Matrix-Tree theorem**

(Koo et al., 2007; D. A. Smith and N. A. Smith, 2007; McDonald and Satta, 2007)

As attention: (Liu and Lapata, 2018)





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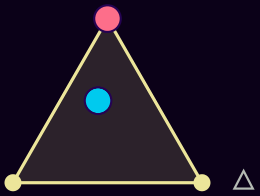
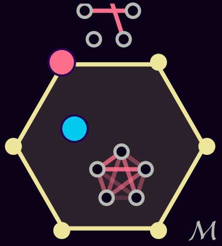
marginals $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$

e.g. matchings → **#P-complete!**

(Taskar, 2004; Valiant, 1979)





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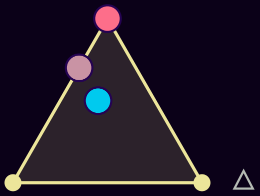

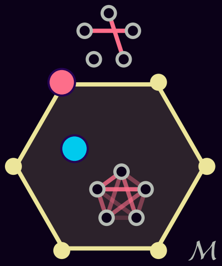

(Niculae, Martins, Blondel, and Cardie, 2018)

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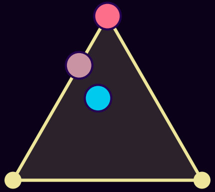
SparseMAP $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$



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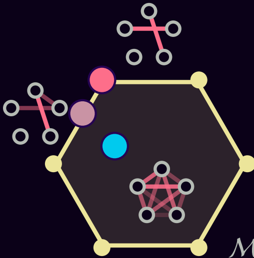

$\mathcal{M}$

SparseMAP Solution

$$\boldsymbol{\mu}^\star = \underset{\boldsymbol{\mu} \in \mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{\eta} - \tfrac{1}{2}\|\boldsymbol{\mu}\|^2$$

=  = .6  + .4 

= $\boldsymbol{A}\boldsymbol{p}^\star$ with very sparse $\boldsymbol{p}^\star \in \triangle^N$

Algorithms for SparseMAP

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Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \operatorname*{arg\,max}_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Algorithms for SparseMAP

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Conditional Gradient

(Frank and Wolfe, 1956; Lacoste-Julien and Jaggi, 2015)

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(Frank and Wolfe, 1956; Lacoste-Julien and Jaggi, 2015)

- select a new corner of $\mathcal{M}$

Algorithms for SparseMAP

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Conditional Gradient

(Frank and Wolfe, 1956; Lacoste-Julien and Jaggi, 2015)

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$$\boldsymbol{a}_{y^\star} = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \underbrace{\left(\boldsymbol{\eta} - \boldsymbol{\mu}^{(t-1)}\right)}_{\tilde{\boldsymbol{\eta}}}$$

Algorithms for SparseMAP

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Conditional Gradient

(Frank and Wolfe, 1956; Lacoste-Julien and Jaggi, 2015)

- select a new corner of $\mathcal{M}$
- update the (sparse) coefficients of $\boldsymbol{p}$
  - Update rules: vanilla, away-step, pairwise

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      (Nocedal and Wright, 1999, Ch. 16.4 & 16.5)
      (Wolfe, 1976; Vinyes and Obozinski, 2017)

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Active Set achieves
finite & **linear** convergence!

Algorithms for SparseMAP

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    (Wolfe, 1976; Vinyes and Obozinski, 2017)

Backward pass

$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

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  - Update rules: vanilla, away-step, pairwise
  - Quadratic objective: **Active Set**
    (Nocedal and Wright, 1999, Ch. 16.4 & 16.5)
    (Wolfe, 1976; Vinyes and Obozinski, 2017)

Backward pass

$\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}$ is sparse

computing $\left(\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\eta}}\right)^\top \boldsymbol{dy}$
takes $\mathcal{O}(\dim(\boldsymbol{\mu})\,\mathrm{nnz}(\boldsymbol{p}^\star))$

Algorithms for SparseMAP

$$\boldsymbol{\mu}^\star = \arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{\eta} - 1/2\|\boldsymbol{\mu}\|^2$$

linear constraints
(alas, exponentially many!)

quadratic objective

Conditi

(Frank and Wolfe, 1956

Completely modular: just add MAP

pass

- select a new corner of $\mathcal{M}$
- update the (sparse) coefficients of $\boldsymbol{p}$
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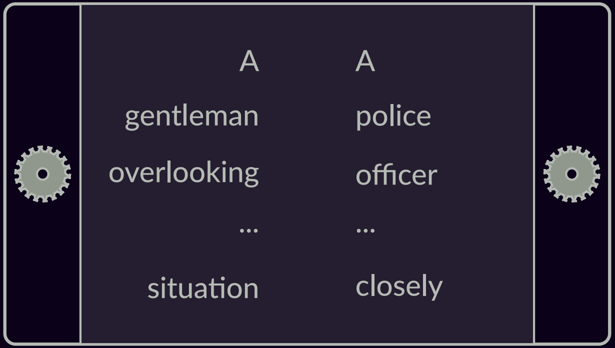
Structured Attention for Alignments

NLI

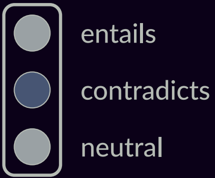
premise: A gentleman overlooking a neighborhood situation.
hypothesis: A police officer watches a situation closely.

input

(P, H)



output



(Model: ESIM (Chen et al., 2017))

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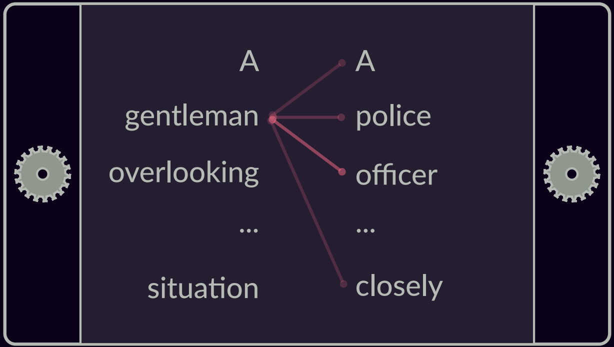

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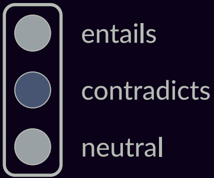

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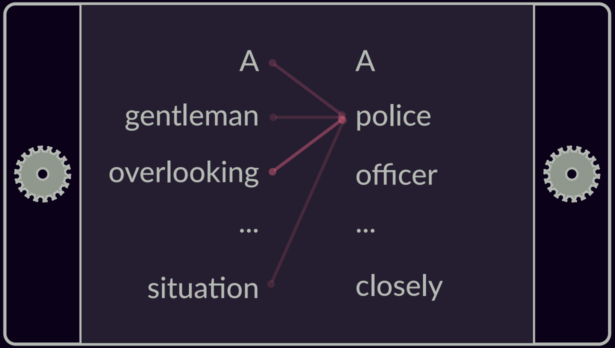
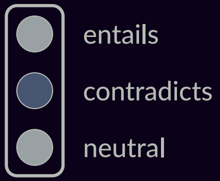


output



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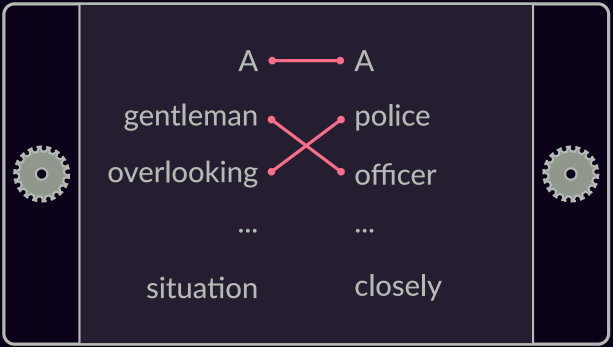
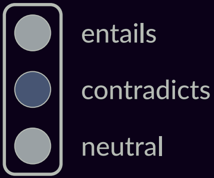


output



(Proposed model: global matching)

In code:

```
# Xp: (n_prem x k)
# Xh: (n_hypo x k)

Z = Xp @ Xh.t()

Up = softmax(Z, dim=1)
Uh = softmax(Z, dim=0)

Xp = cat([Xp, Up     @ Xh])
Xh = cat([Xh, Uh.t() @ Xp])

# ...
```

In code:

```
# Xp: (n_prem x k)
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# ...
```
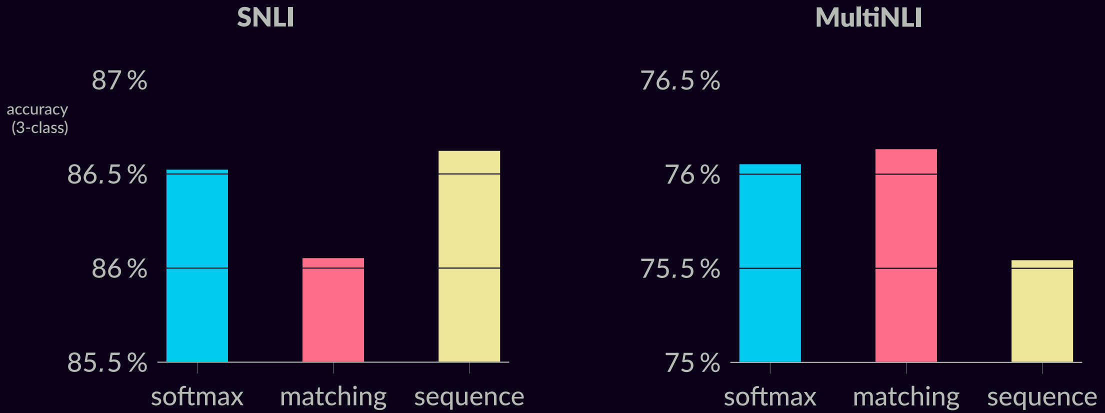
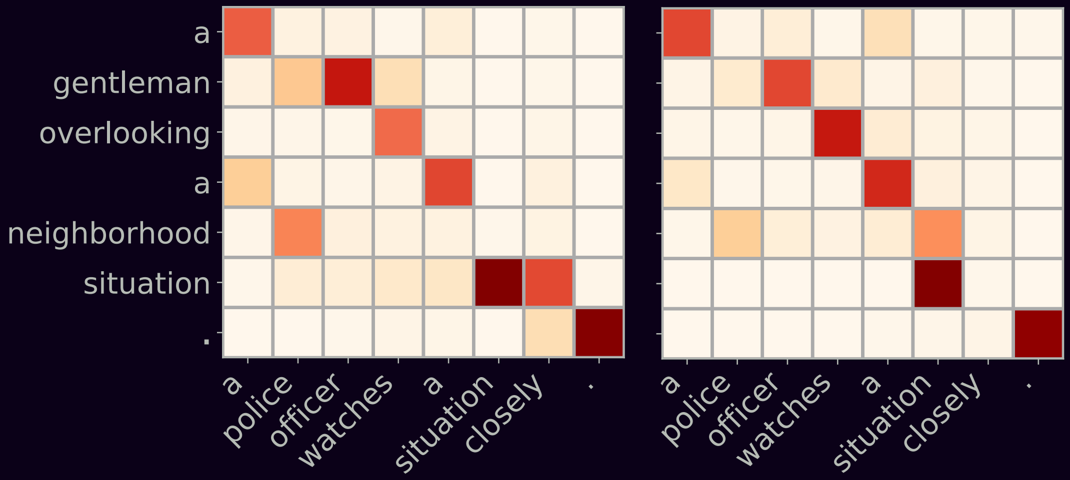

```
# Xp: (n_prem x k)
# Xh: (n_hypo x k)

Z = Xp @ Xh.t()

U = sparsemap_matching(Z)

Xp = cat([Xp, U       @ Xh])
Xh = cat([Xh, U.t() @ Xp])

# ...
```


SNLI
MultiNLI
accuracy
(3-class)
87 %
86.5 %
86 %
85.5 %
76.5 %
76 %
75.5 %
75 %
softmax
matching
sequence
softmax
matching
sequence


a
gentleman
overlooking
a
neighborhood
situation
.
a
police
officer
watches
a
situation
closely
.
a
police
officer
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situation
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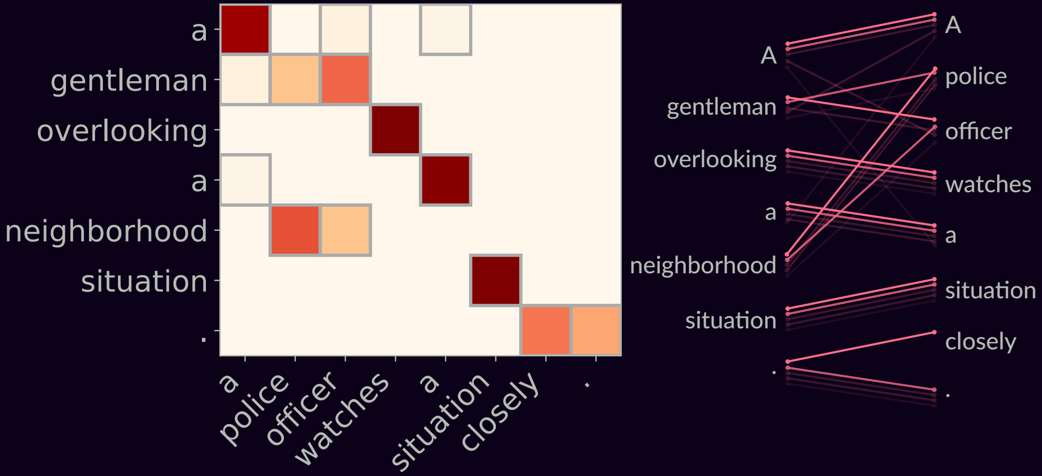
.



a
gentleman
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.

Dynamically inferring the computation graph

Dependency TreeLSTM

(Tai et al., 2015)

closely related to GCNs, e.g.
(Kipf and Welling, 2017)
(Marcheggiani and Titov, 2017)

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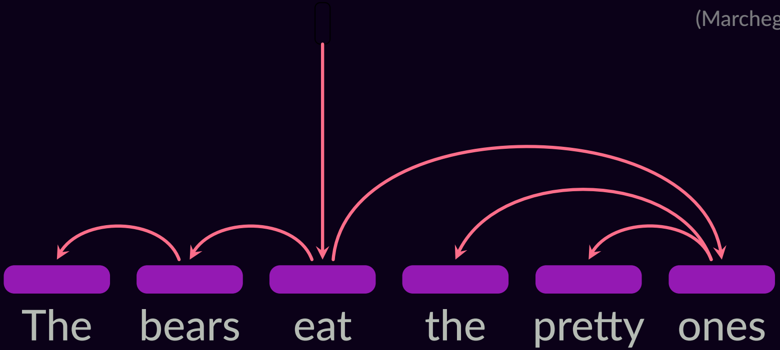

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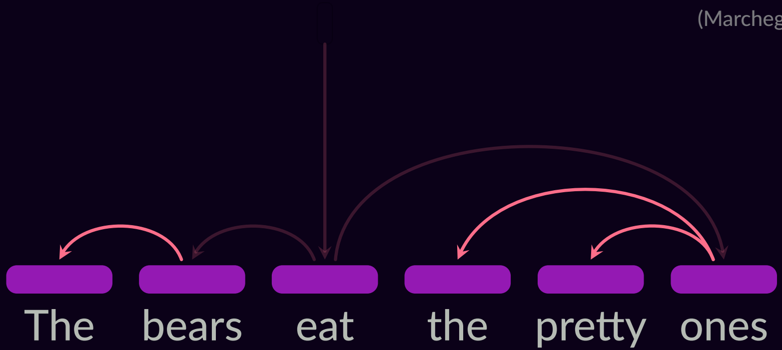

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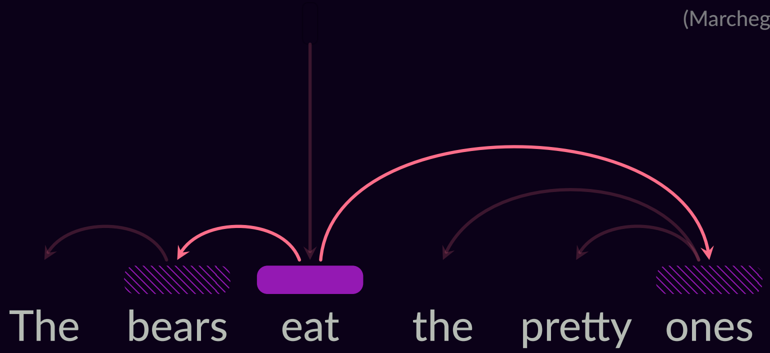

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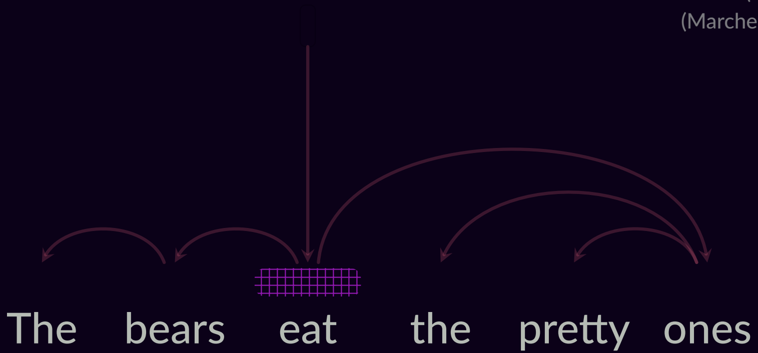

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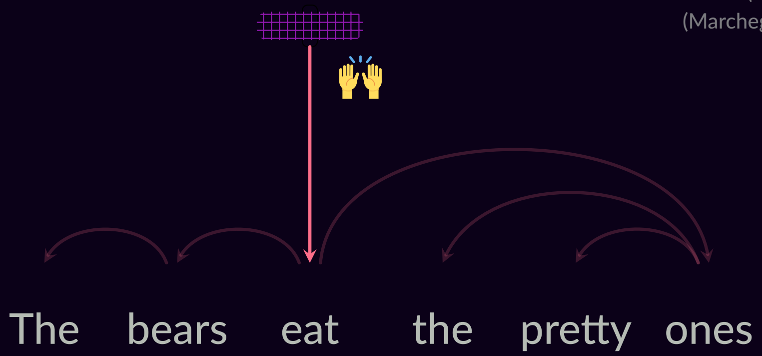
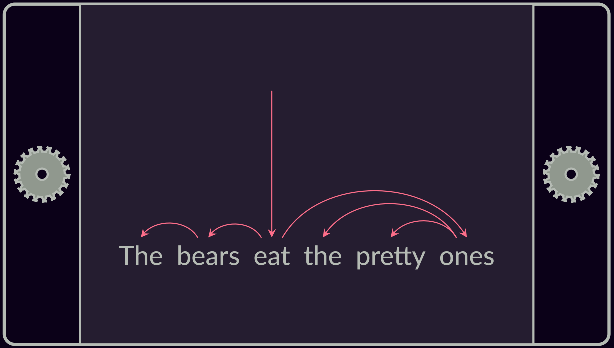


Latent Dependency TreeLSTM

(Niculae, Martins, and Cardie, 2018)

input

x



output

y

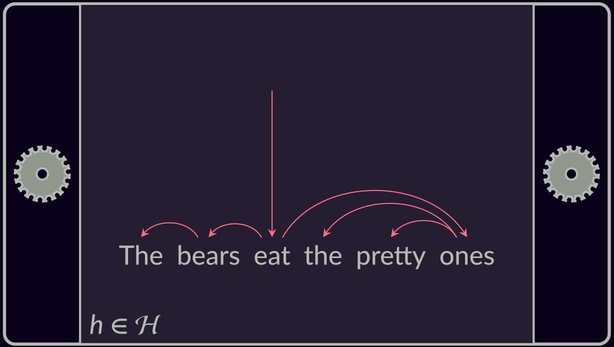
Latent Dependency TreeLSTM

(Niculae, Martins, and Cardie, 2018)

$$p(y|x) = \sum_{h \in \mathcal{H}} p(y \mid h, x)\, p(h \mid x)$$

input

x



output

y

Structured Latent Variable Models

$$p(y \mid x) = \sum_{h \in \mathcal{H}} p \quad (y \mid h, x) \, p \quad (h \mid x)$$

Structured Latent Variable Models

$$p(y \mid x) = \sum_{h \in \mathcal{H}} p_{\boldsymbol{\phi}}(y \mid h, x)\, p_{\boldsymbol{\pi}}(h \mid x)$$

Structured Latent Variable Models

e.g., a TreeLSTM defined by h

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$$p(y \mid x) = \sum_{h \in \mathcal{H}} p_{\boldsymbol{\phi}}(y \mid h, x)\, p_{\boldsymbol{\pi}}(h \mid x)$$

parsing model,
using some $\text{score}_{\boldsymbol{\pi}}(h; x)$

Structured Latent Variable Models

sum over
all possible trees

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Exponentially large sum!

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How to define $\boldsymbol{p_{\pi}}$?

idea 1
idea 2
idea 3

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$$\sum_{h \in \mathcal{H}} \quad \frac{\partial p(y \mid x)}{\partial \boldsymbol{\pi}}$$

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$$\sum_{h \in \mathcal{H}} \quad \frac{\partial p(y \mid x)}{\partial \boldsymbol{\pi}}$$

idea 1 $\quad p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^\star$ else 0 $\qquad$ argmax

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idea 3

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idea 1 $p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^{\star}$ else 0 argmax 😊

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idea 1	$p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^\star$ else 0	argmax	😊	😟
idea 2				
idea 3				

Structured Latent Variable Models

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How to define $p_{\boldsymbol{\pi}}$?

$$\sum_{h \in \mathcal{H}} \qquad \frac{\partial p(y \mid x)}{\partial \boldsymbol{\pi}}$$

😊 😟

idea 1	$p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^\star$ else 0	argmax	😊	😟
idea 2	$p_{\boldsymbol{\pi}}(h \mid x) \propto \exp\big(\mathrm{score}_{\boldsymbol{\pi}}(h; x)\big)$	softmax		
idea 3				

Structured Latent Variable Models

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parsing model,
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How to define $p_{\boldsymbol{\pi}}$?

			$\sum_{h \in \mathcal{H}}$	$\frac{\partial p(y \mid x)}{\partial \boldsymbol{\pi}}$
idea 1	$p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^\star$ else 0	argmax	😊	😟
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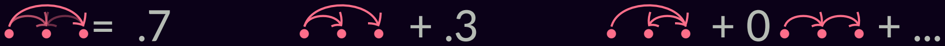
			$\sum_{h \in \mathcal{H}}$	$\frac{\partial p(y \mid x)}{\partial \boldsymbol{\pi}}$
idea 1	$p_{\boldsymbol{\pi}}(h \mid x) = 1$ if $h = h^\star$ else 0	argmax	😊	😟
idea 2	$p_{\boldsymbol{\pi}}(h \mid x) \propto \exp\big(\text{score}_{\boldsymbol{\pi}}(h; x)\big)$	softmax	😱	😊
idea 3		SparseMAP	😊	😊

SparseMAP

 = .7  + .3 

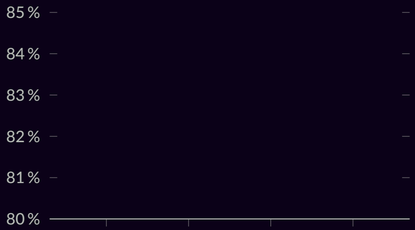
SparseMAP

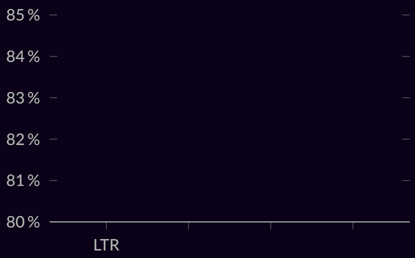


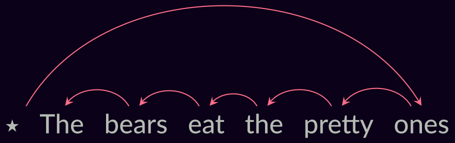
SparseMAP

 $= .7$  $+ .3$  $+ 0$  $+ \ldots$

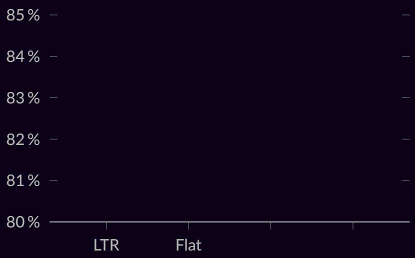
$p(y \mid x) = .7\, p_{\boldsymbol{\phi}}(y \mid$  $) + .3\, p_{\boldsymbol{\phi}}(y \mid$  $)$

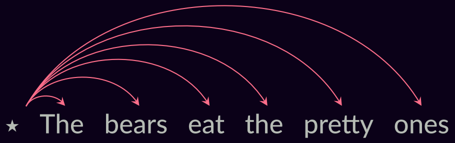

85 %
84 %
83 %
82 %
81 %
80 %



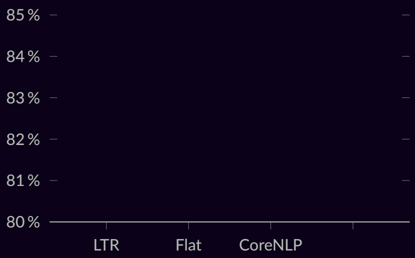


Left-to-right: regular LSTM

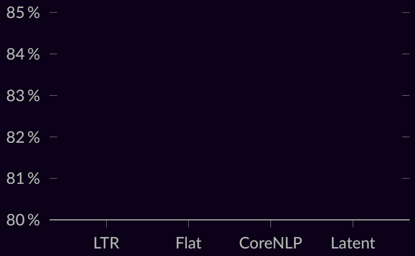




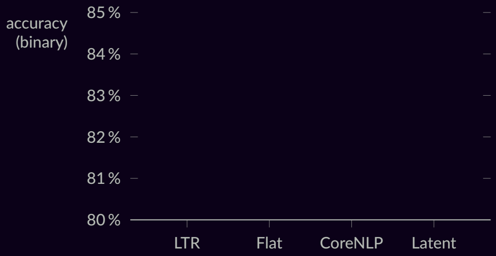
Flat: bag-of-words–like



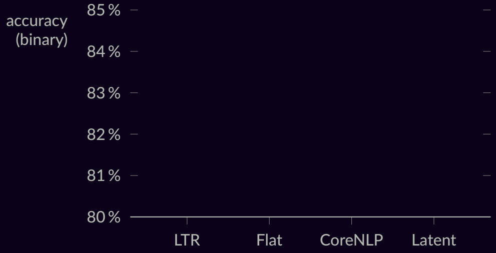
CoreNLP: off-line parser


85 %
84 %
83 %
82 %
81 %
80 %
LTR
Flat
CoreNLP
Latent

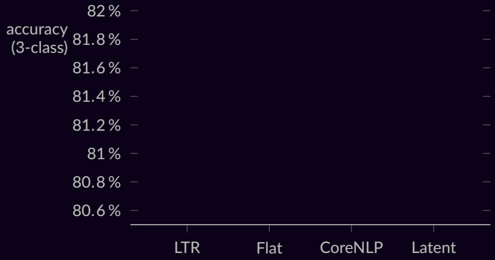
Sentiment classification (SST)



Sentiment classification (SST)



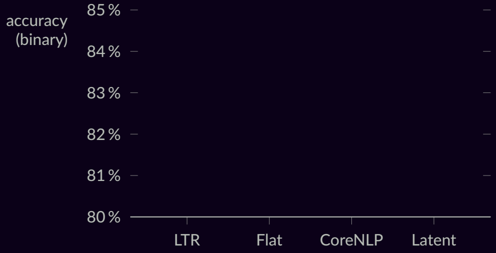
Natural Language Inference (SNLI)



Sentence pair classification (P, H)

$$p(y \mid P, H) = \sum_{h_P \in \mathcal{H}(P)} \sum_{h_H \in \mathcal{H}(H)} p_{\boldsymbol{\phi}}(y \mid h_P, h_H)\, p_{\boldsymbol{\pi}}(h_P \mid P)\, p_{\boldsymbol{\pi}}(h_H \mid H)$$

Sentiment classification (SST)



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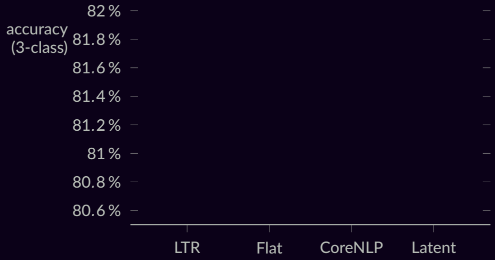
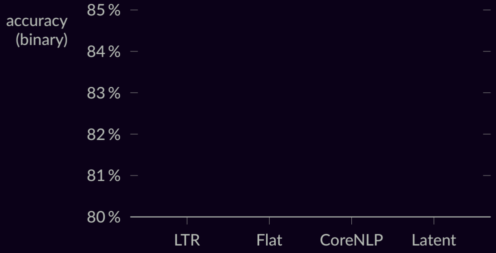


Reverse dictionary lookup

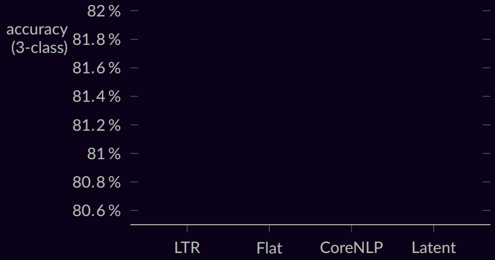
given word description, predict word embedding (Hill et al., 2016)

instead of $p(y \mid x)$, we model $\mathbb{E}_{p_{\boldsymbol{\pi}}}\boldsymbol{g}(x) = \sum_{h \in \mathcal{H}} \boldsymbol{g}(x; h)\, p_{\boldsymbol{\pi}}(h \mid x)$

Sentiment classification (SST)

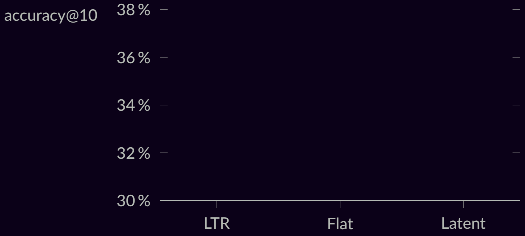


Natural Language Inference (SNLI)

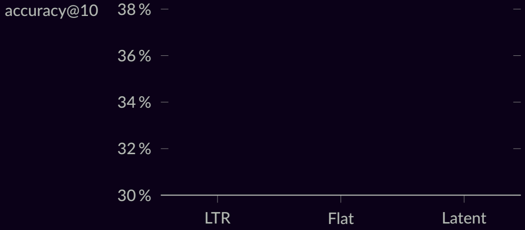


Reverse dictionary lookup

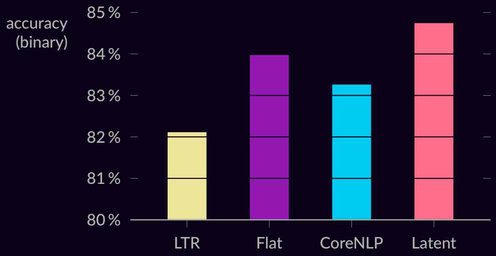
(definitions)



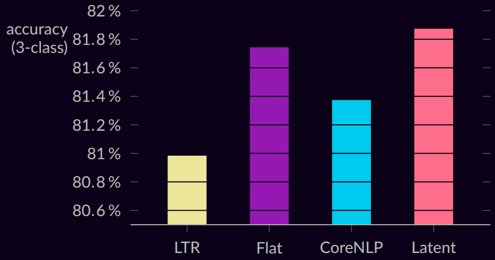
(concepts)



Sentiment classification (SST)

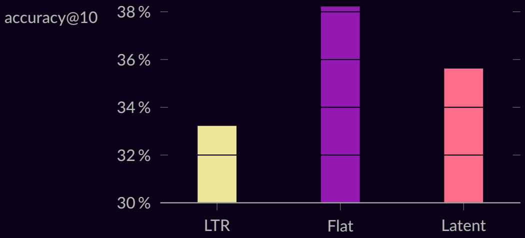


Natural Language Inference (SNLI)

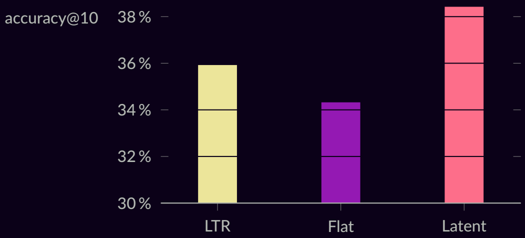


Reverse dictionary lookup

(definitions)



(concepts)



Syntax vs. Composition Order

CoreNLP parse, $p = 21.4\%$

$\star$ lovely and poignant .

Syntax vs. Composition Order

$p = 22.6\%$

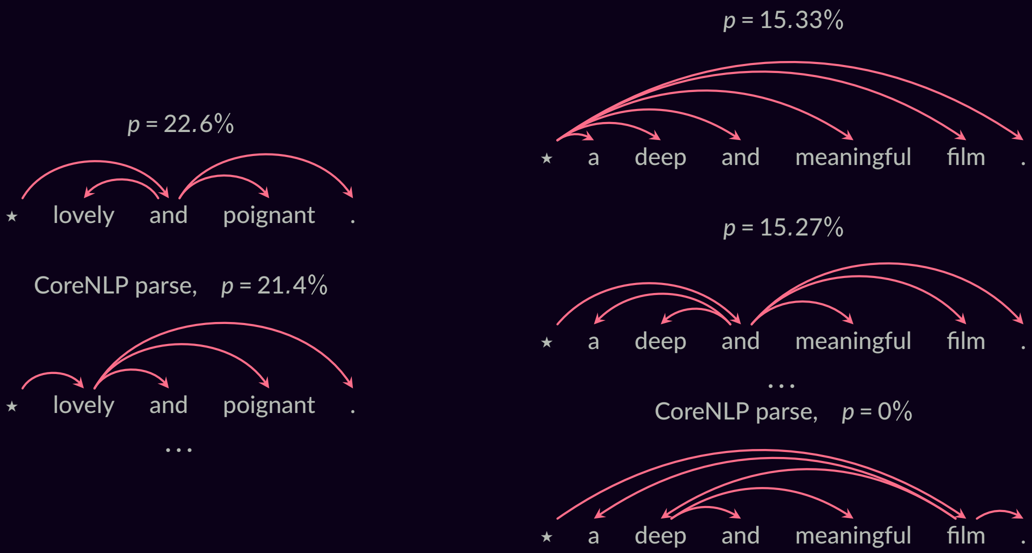
★ lovely and poignant .

CoreNLP parse, $p = 21.4\%$

★ lovely and poignant .

. . .

Syntax vs. Composition Order



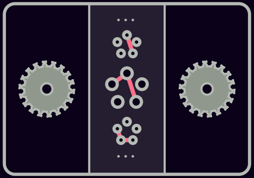
Conclusions

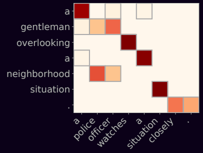
Differentiable & sparse structured inference

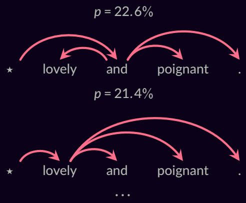
Generic, extensible algorithms

Interpretable structured attention

Dynamically-inferred computation graphs







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github.com/vene/sparsemap
@vnfrombucharest

Extra slides

Acknowledgements

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Some icons by Dave Gandy and Freepik via flaticon.com.

(Danskin, 1966; Prop. B.25 in Bertsekas, 1999)

Danskin's Theorem

Let $\phi : \mathbb{R}^d \times \mathcal{Z} \to \mathbb{R}$, $\mathcal{Z} \subset \mathbb{R}^d$ compact.

$$\partial \max_{\boldsymbol{z} \in \mathcal{Z}} \phi(\boldsymbol{x}, \boldsymbol{z}) = \operatorname{conv}\left\{ \nabla_{\boldsymbol{x}} \phi(\boldsymbol{x}, \boldsymbol{z}^\star) \mid \boldsymbol{z}^\star \in \arg\max_{\boldsymbol{z} \in \mathcal{Z}} \phi(\boldsymbol{x}, \boldsymbol{z}) \right\}.$$

Example: maximum of a vector

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Example: maximum of a vector

$$\begin{aligned}
\partial \max_{j \in [d]} \theta_j &= \partial \max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{\theta} \\
&= \partial \max_{\boldsymbol{p} \in \triangle} \phi(\boldsymbol{p}, \boldsymbol{\theta}) \\
&= \operatorname{conv}\left\{ \nabla_{\boldsymbol{\theta}} \phi(\boldsymbol{p}^\star, \boldsymbol{\theta}) \right\} \\
&= \operatorname{conv}\left\{ \boldsymbol{p}^\star \right\}
\end{aligned}$$

(Danskin, 1966; Prop. B.25 in Bertsekas, 1999)

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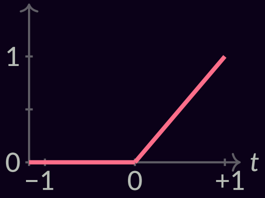
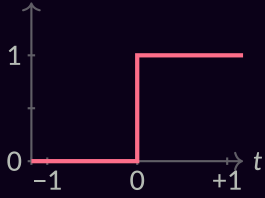
&= \operatorname{conv}\left\{ \boldsymbol{p}^\star \right\}
\end{aligned}$$

$\boldsymbol{\theta} = [t, 0]$



$\max_j \theta_j$



$\{g_1 \mid \boldsymbol{g} \in \partial \max_j \theta_j\}$

Fusedmax

$$\mathrm{fusedmax}(\boldsymbol{\theta}) = \underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top\boldsymbol{\theta} - 1/2\|\boldsymbol{p}\|_2^2 - \sum_{2\le j\le d} |p_j - p_{j-1}|$$

$$= \underset{\boldsymbol{p}\in\triangle}{\arg\min}\, \|\boldsymbol{p} - \boldsymbol{\theta}\|_2^2 + \sum_{2\le j\le d} |p_j - p_{j-1}|$$

$$\mathrm{prox}_{\mathrm{fused}}(\boldsymbol{\theta}) = \underset{\boldsymbol{p}\in\mathbb{R}^d}{\arg\min}\, \|\boldsymbol{p} - \boldsymbol{\theta}\|_2^2 + \sum_{2\le j\le d} |p_j - p_{j-1}|$$

Proposition: $\mathrm{fusedmax}(\boldsymbol{\theta}) = \mathrm{sparsemax}\left(\mathrm{prox}_{\mathrm{fused}}(\boldsymbol{\theta})\right)$

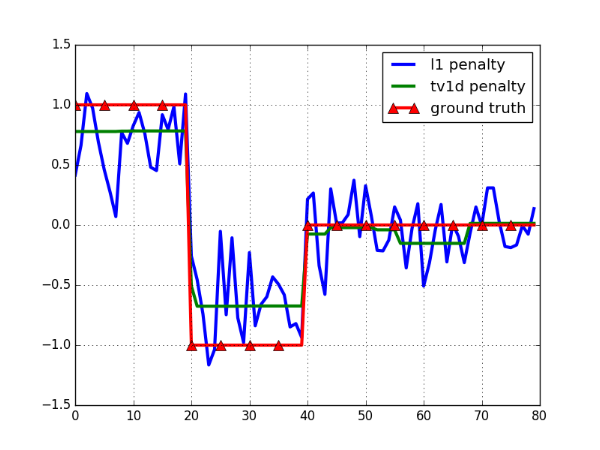
(Niculae and Blondel, 2017)

fusedmax(... $|p_j - p_{j-1}|$

$j-1|$

prox$_{\text{fused}}$(... $j-1|$

Proposi ... $\text{used}(\boldsymbol{\theta}))$

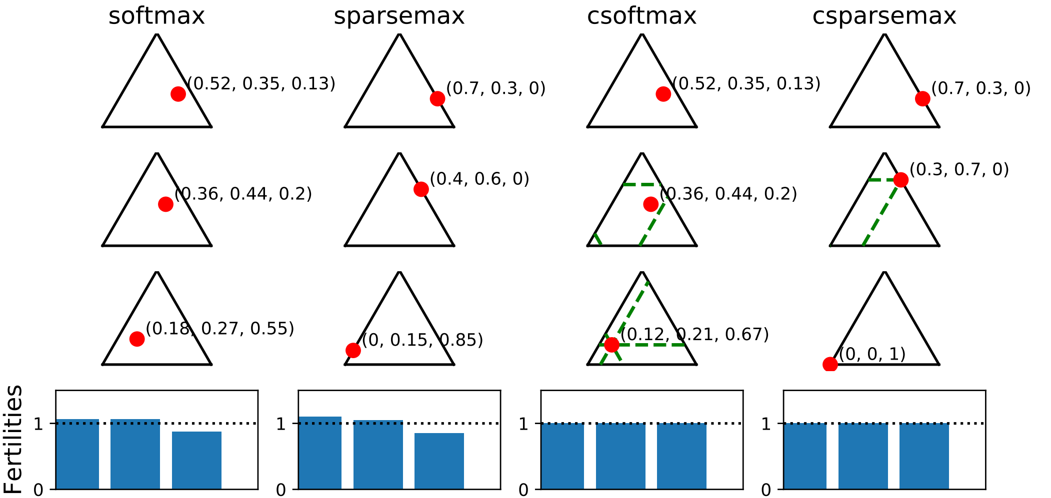


"Fused Lasso" a.k.a. 1-d Total Variation

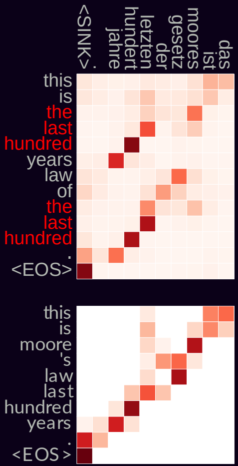
(Tibshirani et al., 2005)

(Niculae and Blondel, 2017)

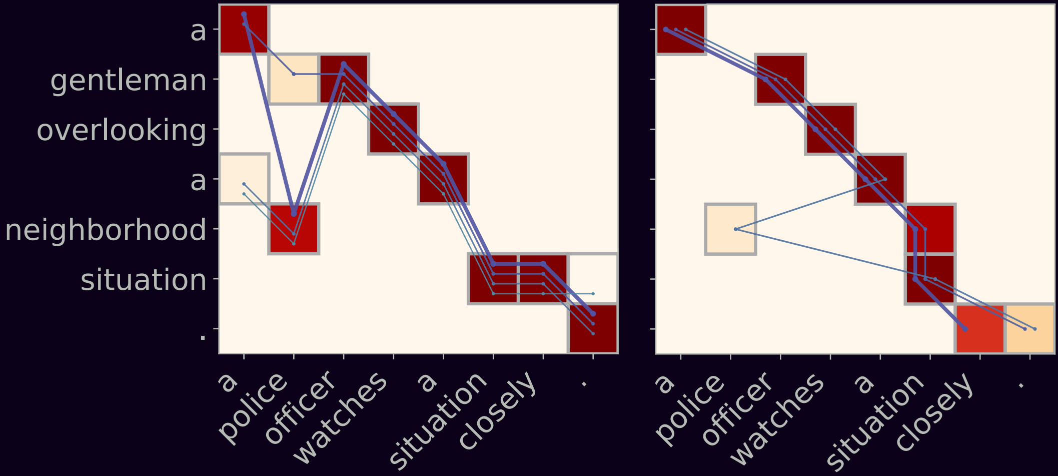
Example: Source Sentence with Three Words



e.g., fertility constraints for NMT



constrained softmax: (Martins and Kreutzer, 2017) **constrained sparsemax:** (Malaviya et al., 2018)


a
gentleman
overlooking
a
neighborhood
situation
.
a
police
officer
watches
a
situation
closely
.

Structured Output Prediction

SparseMAP
$$L_{\boldsymbol{A}}(\boldsymbol{\eta}, \bar{\boldsymbol{\mu}}) = \max_{\boldsymbol{\mu} \in \mathcal{M}} \left\{ \boldsymbol{\eta}^\top \boldsymbol{\mu} - 1/2\|\boldsymbol{\mu}\|^2 \right\} - \boldsymbol{\eta}^\top \bar{\boldsymbol{\mu}} + 1/2\|\bar{\boldsymbol{\mu}}\|^2$$

Instance of a structured Fenchel-Young loss, like CRF, SVM, etc. (Blondel, Martins, and Niculae, 2019)

Structured Output Prediction

SparseMAP

$$L_A(\boldsymbol{\eta}, \bar{\boldsymbol{\mu}}) = \max_{\boldsymbol{\mu} \in \mathcal{M}} \left\{ \boldsymbol{\eta}^\top \boldsymbol{\mu} - {}^1\!/\!{}_2 \|\boldsymbol{\mu}\|^2 \right\} - \boldsymbol{\eta}^\top \bar{\boldsymbol{\mu}} + {}^1\!/\!{}_2 \|\bar{\boldsymbol{\mu}}\|^2$$

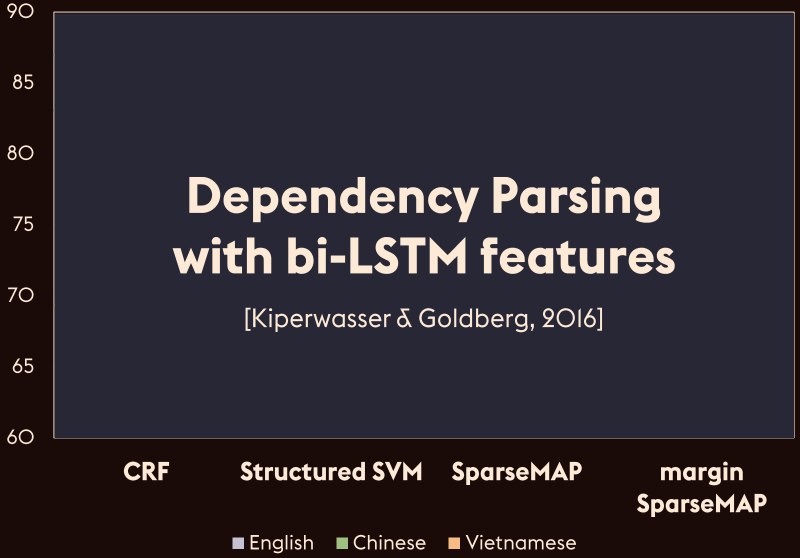
cost-SparseMAP

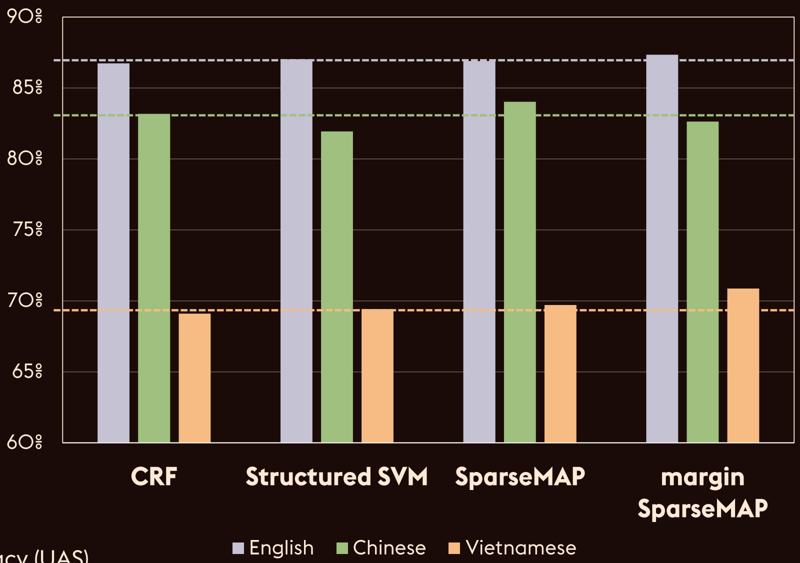
$$L_A^\rho(\boldsymbol{\eta}, \bar{\boldsymbol{\mu}}) = \max_{\boldsymbol{\mu} \in \mathcal{M}} \left\{ \boldsymbol{\eta}^\top \boldsymbol{\mu} - {}^1\!/\!{}_2 \|\boldsymbol{\mu}\|^2 + \rho(\boldsymbol{\mu}, \bar{\boldsymbol{\mu}}) \right\} - \boldsymbol{\eta}^\top \bar{\boldsymbol{\mu}} + {}^1\!/\!{}_2 \|\bar{\boldsymbol{\mu}}\|^2$$

Instance of a structured Fenchel-Young loss, like CRF, SVM, etc. (Blondel, Martins, and Niculae, 2019)

Dependency Parsing with bi-LSTM features

[Kiperwasser & Goldberg, 2016]

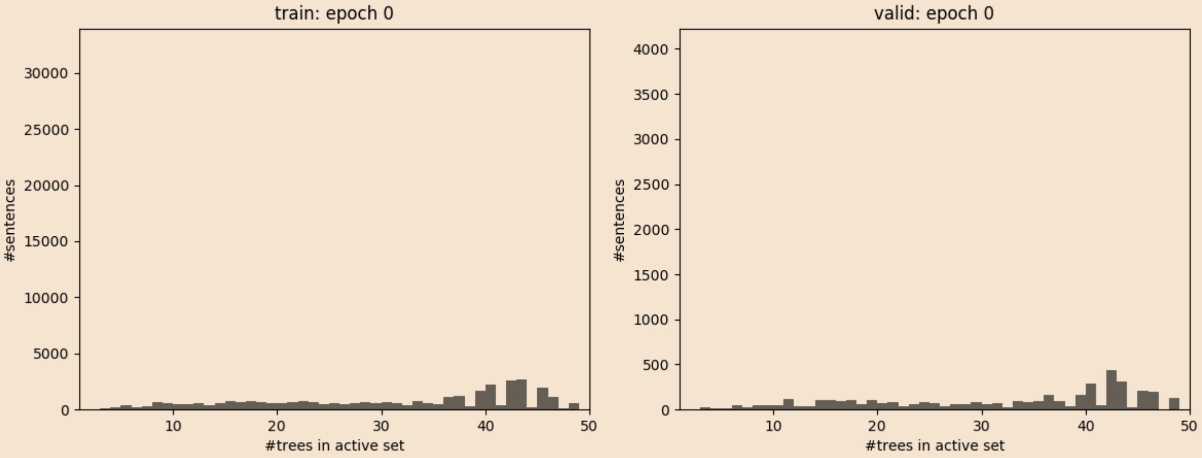




Unlabeled Accuracy (UAS)
Universal Dependencies dataset

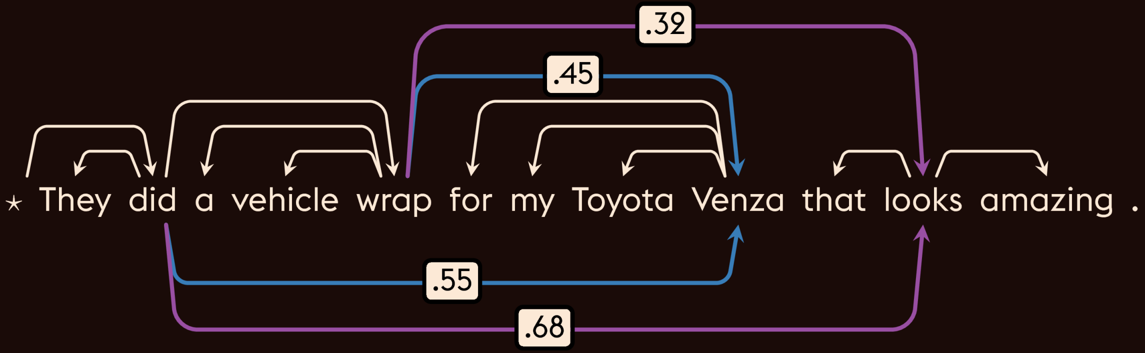
Sparse Structured Output Prediction

As models train, inference gets sparser!



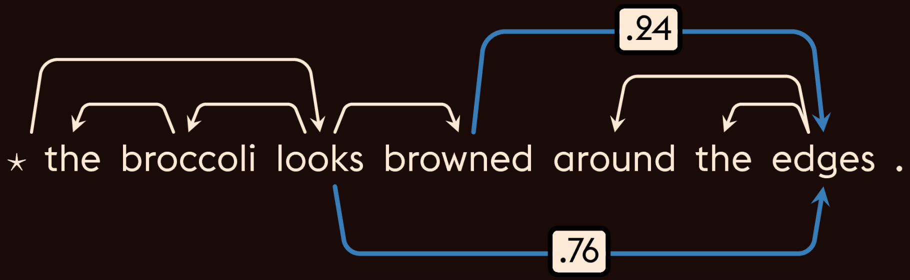
Sparse Structured Output Prediction

Inference captures linguistic ambiguity!



Sparse Structured Output Prediction

Inference captures linguistic ambiguity!



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